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And if it is purely a subjective proof, how can you say the doctrine is proved "in its full scope"? Surely objective applications come within its scope?

It would not be fair in me to ask you to publish my reply to your reviewer's criticisms, though if that reply is justified the criticism must have done the prospects of my book some injury, seeing from what a quarter it comes. But I hope you will see your way to publishing the latter part of this letter in *The Monist*, together with your reply to it, if you think it worthy of such a distinction.

I have just come across, in this month's *Nineteenth Century*, another remarkable instance of reasoning which seems to be rendered entirely nugatory by the want of proper definitions. It is asserted that conceptual thought is impossible without language. At first sight this would certainly appear to be a real assertion. It follows from it that since dogs have no language they have no "conceptual thought." But it may be plainly shown that dogs do entertain "general notions," which in ordinary English would be included under the head of "conceptual thought." The apparent contradiction is however explained when it appears that the author distinguishes general notions as "concepts" or "recepts," according as they *are* or *are not named*. This being his definition of "conceptual thought" as opposed to other thought, it appears that the assertion is only a truism after all, and conveys no real information whatever. To discuss it further is then mere waste of time. The author of the assertion doubtless *wished* it to convey some information, but he did not attend to his definitions and so failed to attain his object.

EDWARD T. DIXON.

MATHEMATICS A DESCRIPTION OF OPERATIONS WITH PURE FORMS.

IN REPLY TO MR. EDWARD DIXON.

It is true, as Mr. Dixon says, that "Any language which seems to imply that there is some dread necessity about mathematical truths is very misleading." But to say, as Mr. Dixon does in another passage, that the truisms of mathematics are arbitrary truths, is more misleading still. The theorems of the formal sciences are not "assertions whose truth depends solely on the definition of their terms." They are "real assertions which convey some real subjective or objective information."

Mr. Dixon objects to Kant's assertion that $7 + 5 = 12$ is not only *a priori* but also synthetic. He declares, in contradistinction to Kant, that it is deduced from definitions alone; that therefore it is empty, and cannot give any information about things. This latter proposition, which is a phrase of Kant's, appears in this context as an inconsistency of Kant's. And it would be an inconsistency, if it had to be understood in the sense in which Mr. Dixon quotes it. We construe Kant's phrase that "the *a priori* is empty, and cannot give information about things," in a different way. We think that Kant intends to say that the *a priori* imparts real information concerning relations and forms; but that it does not impart real information con-

cerning substances or the materiality of things. It is apparent that the assertion $7 + 5 = 12$ cannot be derived from the definitions of 7 or 5. Similarly, the ideas of higher mathematics are not deduced from the few definitions of elementary mathematics that tell what points, lines, parallels, etc., are. Of what, then, are these complex theorems of mathematics, products? They are not derived from sense-experience, nor from the definitions of their ultimate elements.

Is their origin mysterious? Here Kant leaves us in the lurch; he simply declares that formal truth is *a priori* and transcendental; and those of his disciples who call themselves, with preference, transcendentalists, have ample occasion to introduce in this *lacuna* of Kantian thought, all the mysticism they please.

The problem of the origin of the truths of formal sciences is not so difficult as it is sometimes represented. The theorems of higher mathematics are the products of certain *operations* performed with the elementary forms described in the definition with which the mathematician starts. These operations are not arbitrary; they are not merely verbal processes; they are realities of highest importance. Not material realities, but realities, nevertheless. They are functions, and mathematics deals with the products of functions. It is true that we might call twice two by any other name than four; we might call it *vier*, or *quatre*, but the operation $1 + 1 + 1 + 1$ would remain the same, by whatever name we call its result. Mathematical truths, accordingly, are not empty in the sense that they are meaningless; for they are significant in the highest degree. They give real information, not about things, but about certain relations that obtain among things. They describe certain operations in which formal relations are traced. And they describe them exhaustively, so that the result is, as the Germans call it, *eindeutig bestimmt*, and the result will, under all circumstances, be the same. Twice two *will always be* the same as $1 + 1 + 1 + 1$. This "it will always be," is called necessary. There is nothing dreadful about it, nor is there any mystery connected with it. It is not an awful fate that decrees it, but it is the nature of sameness, that the same is and will be the same, so long as it remains the same.

It is often overlooked that every number in arithmetic is the result of an operation which is symbolised by a certain figure. Numbers are not concrete things; and as soon as we forget that they are products of a function, we are liable to lapse into mistakes. This happens most frequently with the numbers "zero" and "infinite." The latter of these two symbols is often looked upon as a concrete thing; and because the infinite, with actual reality, is, in its completeness, inconceivable, it has made, of every one who stumbled over this stone of offence, a mystic, and many a radical, fearless thinker bows down to worship before the idea of infinitude-function as it would be if it were a real thing.

Says Mr. Dixon, "Our reason cannot inform us about the form of existence, unless it is first given." This is very true. The form is given, and formal systems such as the numerical system and the lines and figures of mathematics are mental constructions built of the stones quarried out of the relational given in experience.

Form being given, we can reason about the form of existence in general. We can have ready in our minds systems of pure forms to apply to all the various cases of our experience. And this will help us in unravelling the problems of reality, and in extending our knowledge in those fields with which we are little acquainted. Far be it from us to consider the definitions, the operations, and the results of the formal sciences as purely verbal; if they were, mathematics would lose all the great importance which it undeniably possesses, and become mere verbiage.

I confess that I do not understand Mr. Dixon when he says: "It is only because our reasoning faculties are limited that symbolic arguments are necessary at all." In my opinion all mental activity is symbolic. Every idea is a symbol that signifies something. It is not because our reasoning faculties are limited that symbolic arguments are necessary at all, but symbolism is the nature of our mind, and symbols are the elements with which our reasoning faculties have to deal. In this sense, every argument is symbolic. If it symbolises sense-experience, it represents our knowledge of what may be called the materiality of things. If it symbolises operations with pure forms, it represents the purely formal relations of mathematics logic, algebra, etc.

The doctrine of the conservation of matter and energy in reality means nothing more or less than that there is no increase or decrease in the world at large. Nothing originates out of, and nothing disappears into, nothing. It means that twice $1 + 1$ is $1 + 1 + 1 + 1$, neither more nor less; or, in other words, it means that all events are transformations. New things originate, but their newness consists in their forms. In this sense the law of the conservation of matter and energy would have to be called, from Mr. Dixon's standpoint, other differences neglected, a truism. It is not a truism, in the sense of being arbitrary, but in the sense of being a purely formal truth, as are all mathematical theorems.

Mr. Dixon refers to a passage in *Fundamental Problems*, in which I say that "To four-dimensional beings, Kepler's third law would most probably appear as the cubes of the times of revolutions of the planets being proportional to their mean distances to the fourth power." His questions, "What sort of an assertion do you take Kepler's law to be?" and "Why should the result appear different to them?" show that Mr. Dixon has overlooked the condition on which this proposition was made. The first sentence of this paragraph begins with the words, "If space inhered, as Kant maintains, in the thinking subject only, spatial relations and laws would appear different to four-dimensional beings." Space relations are not subjective, in my opinion, but objective. Therefore, since space relations do not inhere in the thinking subject only, because they are a feature of the objective world, and inhere in the thinking subject in so far as it is at the same time an object in the objective world, Kepler's law would appear to four-dimensional beings, if they could exist at all, just the same as it does to us three-dimensional beings.

P. C.